Joint Energy and Reserves Auction with Opportunity Cost Payment for Reserves

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***Abstract*— System operators in the electricity industry are required to procure reserve capacity to deal with unanticipated outages, demand shocks, and transmission constraints. One traditional method of procuring reserves is through a separate capacity auction with two-part bids. We analyze an alternative scheme whereby reserves are procured through the energy market using only energy bids, and capacity payments are made based on a generator's implied opportunity cost. By using the revelation principle, we are able to derive the equilibrium bidding function in this market and show that generators have a clear incentive to understate their costs in order to capture higher capacity rents. We then show that in spite of making energy payments based on the marginally *procured* unit, the expected energy costs under our scheme are bounded by that of a disjoint auction. We then give a numerical example for a special case of uniform demand distributions.**

I. Introduction

A common feature of restructured electricity markets is that an Independent System Operator (ISO) is charged with the task of maintaining reliability of the electricity network in real time. Typically the ISO will perform this by procuring electricity reserves in advance, which can then be quickly dispatched to maintain system reliability in real-time.

In competitive markets, the assignment of generating units to reserve status is done through some form of market mechanism. Traditionally, the ISO will run a reserve auction which is separate from any other energy markets it operates. Under this scheme, it will normally solicit a two-part bid from each generator—a capacity and energy price. The ISO will then compare all the bids by using some scoring rule, and based on that make assignment and dispatch decisions. Units which are assigned reserve status receive a capacity payment, regardless of whether or not they are actually called to generate energy *ex post*. Units which are dispatched to generate in real-time are given a supplemental energy payment.

The market design challenge is to devise the scoring and settlement rule in such a way so as to prevent generators from collecting excessive rents by gaming the market. A well known procurement auction of this sort which highlights the dangers of a poorly-designed mechanism were California's 1993 round of biennial resource planning update (BRPU) auctions. The mechanism was designed to resemble a Vickery auction [1] whereby the bidder with the lowest score in the initial auction was allowed to negotiate terms for a contract similar to those offered by the bidder with the second-lowest score. The rationale for this auction mechanism was that because of its second-price nature, generators would be inclined to bid their true costs. Bushnell and Oren [2] [3] predicted that the specific scoring rule used in that auction would lead to an understatement of marginal costs, which turned out to be true.

To deal with this incentive problem, Bushnell and Oren [3] devise a discriminatory pricing and settlement rule. They show that in their auction, generators will reveal their true

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costs so long as they agree with the ISO on the probability distribution of energy calls. Chao and Wilson [4] devise an alternative scheme which is based on a uniform settlement price, and show that truthful revelation of costs is incentive compatible under that settlement scheme as well. Furthermore, they point out that their design is more robust in the sense that it does not require the ISO and generators to agree on the probability distribution of dispatched energy. In contrast to these separate two-dimensional procurement auctions which have been analyzed in the past, we consider a reserve auction which is integrated with the day-ahead market and based solely on energy bids. Assignment to reserve status and subsequent dispatch is done based on the merit order of those energy bids. Generators which are dispatched to generate receive a uniform market clearing price for energy. Those which are held for reserves but not dispatched receive a capacity payment based on their implied opportunity cost, which is the difference between the uniform market-clearing price for energy and their own energy bid.

The main goal of this paper is to model the integrated market for energy reserves, and to derive the equilibrium bidding behavior of generators. The remainder of this paper is organized as follows. Section II presents a formulation for the market and derives the equilibrium bidding strategy of generators. In Section III we derive a bound on the expected energy payments. We then analyze bidding behavior with a numerical example in Section IV. Section V concludes.

II. Bidding In The Reserve Market

We propose running a combined day-ahead market for energy and reserves. The ISO will procure reserve energy from this market based on its estimation of how much will be necessary to meet the next day's load reliably. Of these procured reserves, some will be dispatched to generate energy, which depends on the actual real-time load. Dispatched load will be paid a uniform market clearing price, and generation capacity which is procured but not dispatched will be given a capacity payment based on its opportunity cost. The chart in Figure 1 illustrates how the proposed market would settle for a given procurement and dispatch quantity.

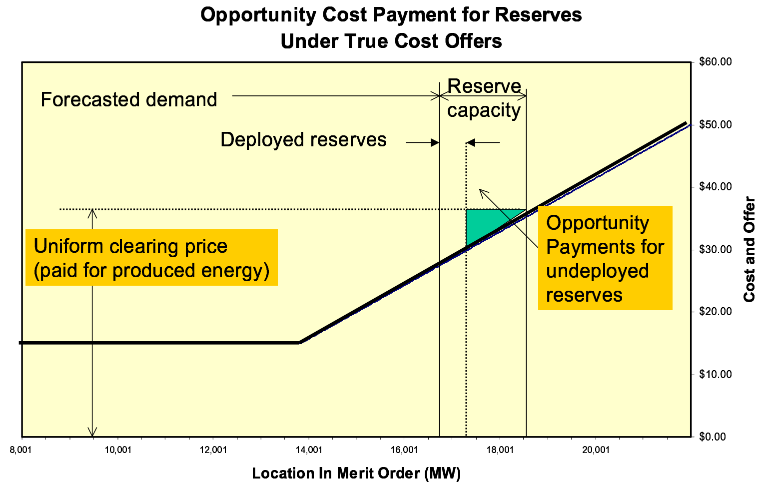

Fig. 1. Market Settlement Example.

A. Assumptions

We assume that the amount of energy procured will be a random variable, Q, which has an atomless distribution function $F(Q)$ with a non-zero density on its support of $[q_0, q_1]$. One may think of this procurement quantity as some forecast of the load for the next day, plus a reserve margin which is an additional $r\%$ of that forecast quantity.

In real-time, a certain fraction, m, of that procured quantity will be dispatched to generate energy. Again, we assume this fraction m to be random and to have a distribution function $H(m)$, with a support of $[\beta, 1]$. If we take the view that the procured quantity Q is the forecasted load plus a reverse margin of $r\%$, then we would define $\beta = \frac{1}{1+r}$, and the actual fraction dispatched will be somewhere between the forecast quantity and the forecast plus the reserve margin. Note that since m has an upper support of 1, we implicitly assume the ISO will never have a shortfall of procured resources.

Naturally, we can define $E = m \times Q$ to be the total capacity dispatched for energy. The dispatch quantity, E, will also be stochastic and its distribution function, $G(E)$, will be implied by $F(\cdot)$ and $H(\cdot)$ and will have a support of $[\beta q_0, q_1]$. Finally, in our analysis of the opportunity cost auction, we will ignore network effects. This is equivalent to assuming there is no network congestion. We also ignore the possibility of different ramp rates among generators so that energy is always dispatched in merit order based on marginal cost. In a realistic setting where generators have different ramp rates the ISO may dispatch an 'out of merit' expensive but slow-responding generator for energy before a low cost but fast-responding unit in order to save the fast response unit for reserves in case of an emergency.

As for the generators, we assume they are risk-neutral profit-maximizing firms and that each MW of generating capacity, which is characterized by its location within the resource stack q, is bid individually of others (i.e. there are no multiunit effects). Generators have perfect information regarding the aggregate cost function, $c(q)$, where q defines the location of each MW within the resource stack, along with their own position in the merit order. Using this information, generators will submit energy bids for each incremental MW of generation. The ISO will then procure capacity day-ahead based on the merit order of the energy bids. All generators which are called to generate in real-time will be paid a uniform market-clearing price which is the bid of the marginal *procured* (not dispatched) unit. Generators which are procured but not dispatched will receive their implied opportunity cost of being held for reserve, which is the difference between the market clearing price and their own bid.

B. Derivation Of Equilibrium Bidding Function

We theorize that due to the opportunity cost based capacity payment used in this market, generators will have an incentive to shade their bids below cost in order to capture capacity rents. To derive the equilibrium bidding function of the generators, we use the condition that each generator is maximizing expected profits. Suppose that all generators bid according to a monotonically-increasing bid function, $b(q)$.[1] An arbitrary generator located at q within the resource stack must choose a bid $\hat{b}$ to maximize its expected profits given the bidding behavior of the other generators. By appealing to the revelation principle, we can restrict attention to a direct revelation mechanism, wherein the generator reveals a location within the resource stack. Thus, if we let $\hat{q} = b^{-1}(\hat{b})$, the generator's bid of $\hat{b}$ is equivalent to it revealing a location $\hat{q}$ within the resource stack. We can then express the generator's expected profits as a function of its actual (q) and revealed ($\hat{q}$) location within the stack:

$$\pi^e(\hat{q}, q) = \int_{\hat{q}}^{+\infty} [b(x) - b(\hat{q})] dF(x) + [b(\hat{q}) - c(q)] \times [1 - G(\hat{q})] \tag{1}$$

Differentiating equation (1) with respect to $\hat{q}$ gives the first-order necessary condition (FONC) for optimality of the bid choice $\hat{q}$, which is:

$$\frac{\partial}{\partial \hat{q}} \pi^e(\hat{q}, q) = -b(\hat{q})g(\hat{q}) + \frac{db(\hat{q})}{d\hat{q}}[F(\hat{q}) - G(\hat{q})] + c(q)g(\hat{q}) = 0$$

Since this is a truthful revelation mechanism, we let $\hat{q} = q$ which yields the differential equation:

$$\frac{db(q)}{dq} = \frac{[c(q) - b(q)]g(q)}{G(q) - F(q)} \tag{2}$$

with the boundary condition,

$$b(q) = c(q) \text{ for } q \text{ s.t. } G(q) = F(q).$$

Thus, the optimal bidding behavior of the generators will be dictated by the differential equation (2). Note that if $G(q) \geq F(q)$, then $b(q) \leq c(q) \iff \frac{db(q)}{dq} \geq 0$. Because we assume $m \leq 1$, it is clear that $G(q) \geq F(q)$. For an intuitive explanation of this condition, note that it is equivalent to $1 - G(q) \leq 1 - F(q)$ which says that for any quantity $\hat{q}$ there is a higher probability of having to procure at least $\hat{q}$ MW than having to actually dispatch at least $\hat{q}$ MW.

III. Expected Energy Cost

An important policy question when designing a market is how the expected procurement costs will compare to alternative designs. The standard design, which we use as our benchmark, is a disjoint market for energy and reserves. This comparison is slightly confounded by the fact the cost of reserving a unit can be difficult to ascertain. Indeed, our model assumes no direct cost of reserving capacity, thus the only economic cost of being assigned reserve status is the opportunity cost of not selling in the energy market—which is the basis of our settlement scheme. Thus our comparison will be based on expected energy costs.

A standard criticism of using an opportunity-cost based settlement rule in our market is that because energy payments are based on the marginally *procured* (as opposed to dispatched) unit, it overcompensates energy producers. Although this pricing rule could result in overcompensation of energy, we find that the equilibrium bid-shading behavior

[1] The monotonicity requirement is needed so the bid function preserves the merit order of the generators.

will actually mitigate such overpayments. To see this, we first study a general class of auction mechanisms, in which our auction falls. For the auctions we analyze, we assume (in addition to the assumptions of Section II):

1) A generator is dispatched to generate energy based only on whether realized demand is greater than her revealed location, i.e. $E \geq \hat{q}$,
2) Generators which are dispatched are paid a uniform price based on E, the demand realization and m, the fraction of reserves dispatched.[2]

We can now show, using a technique similar to that used by Riley and Samuelson [5], that under any auction mechanism with a settlement rule meeting these assumptions, expected generator profits are equivalent.

Theorem 3.1: Suppose the stated assumptions hold and generators are risk-neutral profit maximizers. The first-best equilibrium bidding strategy for any auction rule will yield a generator located at q within the resource stack an expected profit of

$$\int_q^{+\infty} c(E)dG(E) - c(q)[1 - G(q)]$$

Proof: By assumption, generators dispatched to produce energy receive a uniform payment which is a function of E and m. Suppose $P(E, m)$ is a function, giving the payment for each realization of E and m. We can then write the expected profit of a generator located at q and revealing a location $\hat{q}$ as:

$$\pi^e(\hat{q}, q) = \int_\beta^1 \int_{\hat{q}}^{+\infty} [P(E, m) - c(q)]dG(E)dH(m) \quad (3)$$

The FONC for maximizing equation (3) is:

$$\frac{\partial}{\partial \hat{q}} \pi^e(\hat{q}, q) = g(\hat{q}) \int_\beta^1 [c(q) - P(\hat{q}, m)] \cdot dH(m) = 0$$

Assuming a truthful revelation mechanism, this becomes:

$$\begin{aligned} \int_\beta^1 [c(q) - P(q, m)] \cdot dH(m) &= 0 \\ \int_\beta^1 P(q, m) \cdot dH(m) &= c(q) \quad (4) \end{aligned}$$

[2]Note that this assumption allows payments to also depend on the reserved quantity, $Q = E/m$.

Substituting equation (4) into equation (3) yields:

$$\begin{aligned} \pi^e(q, q) &= \int_q^{+\infty} [c(E) - c(q)] \cdot dG(E) \\ &= \int_q^{+\infty} c(E)dG(E) - c(q)[1 - G(q)], \end{aligned}$$

which is the desired expression. ■

We can now look at the two settlement rules, in which energy payments are based on the marginally dispatched and procured units.

- **Marginally Dispatched Payments**
  When payments are based on the marginally dispatched unit, our payment function is:
  $$P(E, m) = b(E)$$
  the bid of the last dispatched unit. Substituting this into equation (4) gives us:
  $$b(q) = c(q).$$
  Thus, generators bid their true cost of generation.
- **Marginally Procured Payments**
  When payments are, instead, based on the marginally procured unit, the payment function becomes
  $$P(E, m) = b(E/m)$$
  When we substitute this into equation (4), we can derive the following integral expression characterizing the optimal bid function:
  $$\int_\beta^1 b(q/m)dH(m) = c(q).$$

The result of Theorem 3.1 (as demonstrated by our two examples) relies on generators optimally adjusting their bids to maximize profits under any given settlement rule. As a result, their expected payments are exactly equivalent to what they would acheive under an auction which pays them based on the marginally dispatched unit.

As a simple corollary, any other bid function will give expected profits no greater than what can be achieved with the optimal bid function. In general, the bid function in our auction with opportunity cost capacity payments will result in lower energy payments, because generators shade their bids to receive greater capacity rents.

IV. Numerical Example

In order to fully derive equilibrium bidding behavior in this market, we must make assumptions on the two demand distributions and generator costs. We will now study an example in which the two distribution functions $F(\cdot)$ and $H(\cdot)$ are uniform.

A. Bidding In Uniform Distributions Case

In our example, we assume the density of the procurement quantity will be:

$$f(Q) = \frac{1}{q_1 - q_0},$$

and likewise the density of the dispatch quantity will be:

$$h(m) = \frac{1}{1-\beta}.$$

These two distribution functions imply the density of the dispatched quantity, which is:

$$g(E) = \begin{cases} \frac{1}{(1-\beta)(q_1-q_0)} \log(\frac{E}{\beta r_0}) & \text{for } \beta q_0 \leq E \leq q_0, \\ \frac{1}{(1-\beta)(q_1-q_0)} \log(\frac{1}{\beta}) & \text{for } q_0 \leq E \leq \beta q_1, \\ \frac{1}{(1-\beta)(q_1-q_0)} \log(\frac{r_1}{E}) & \text{for } \beta q_1 \leq E \leq q_1. \end{cases}$$

Using these density functions, we can derive the equilibrium bidding behavior in the market numerically[3]. In our example we assume values of $q_0 = 16,500$MW and $q_1 = 22,000$MW, a reserve margin of 10% which corresponds to $\beta \approx 0.9091$, and a linear cost function capped from below at \$15 per MWh. The chart in Figure 2 shows the equilibrium bidding strategies in relation to the cost of each unit. We see that generators which have a positive probability of being procured for reserves but not dispatched to generate energy have an incentive to shade their bids below true cost. This is due to two effects which confound one another. As a generator moves further up in the resource stack, its probability of being dispatched (or procured) falls. Thus there is a lower chance of it being forced to generate at a loss. Furthermore, when a generator is procured day-ahead, its probability of being dispatched for energy depends on how close it is to setting the market-clearing price. In other words, when the market clearing price is close to its own bid, then the probability of being dispatched is relatively low. It is only as the marginal unit is further up the resource stack (meaning the market clearing price is higher) that the probability of dispatch rises.

[3]We are unable to derive a closed-form solution to the differential equations.

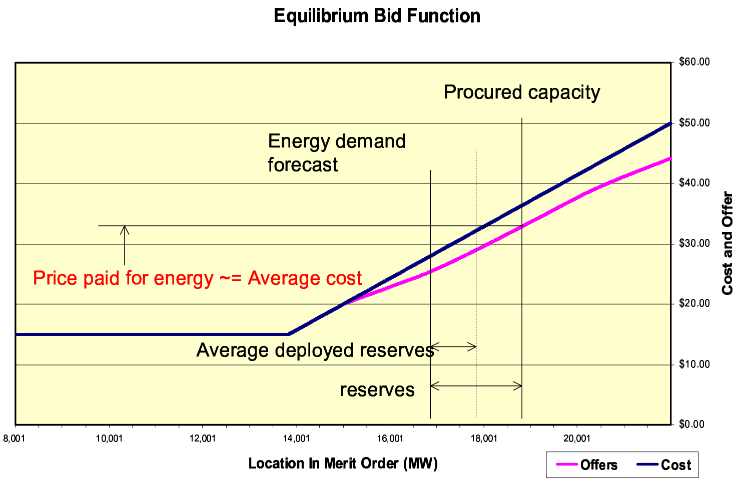

Fig. 2. Equilibrium Bidding Function: Uniform Example.

B. Expected Procurement Costs

Using the data from our numerical example, we can calculate expected energy costs under our auction mechanism and compare it to what it would be in an auction which pays based on the marginally dispatched unit. Theorem 3.1 tells us that expected energy profits under our auction design can be no greater than what would be achieved when energy payments are based on the marginally dispatched unit.

We can numerically compare the expected costs to see that this is indeed true. The expected energy costs when generators bid according to our equilibrium bid function and energy payments are made based on the marginally procured unit is:

$$K_B^e = \int_{q_0}^{q_1} (\int_{\beta p}^{p} s \cdot b(p) dG(s|Q=p)) dF(p).$$

Whereas, if generators bid true cost in an auction which pays based on the marginally dispatched unit, the expected cost will be:

$$K_C^e = \int_{\beta q_0}^{q_1} s \cdot c(s) dG(s).$$

Using the data from our example, we find that under our proposed scheme dispatch costs would be approximately \$655,586, whereas under truthful revelation and payments based on the marginally dispatched unit, expected energy costs will be approximately \$664,942. Figure 3 illustrates the difference in the settlement price when energy is paid the true marginal cost of the dispatched unit, as opposed to the shaded bid of the marginally procured unit under the proposed auction.

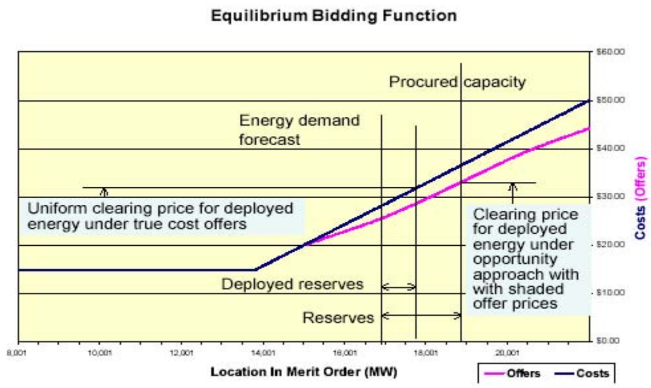

Fig. 3. Cost Comparison Example.

V. Concluding Remarks

We have shown that one viable alternative to the standard two-dimensional procurement auction for reserves is to conduct the procurement auction within the day-ahead energy market itself. By procuring excess capacity day-ahead and dispatching whatever resources are necessary in real-time, the system operator can run a single transparent market as opposed to two separate ones which is a standard design in use today. A clear advantage of this is that generators no longer have to decide which market to bid into, which can be an issue if the two are operated simultaneously. The fact that generators bid according to a monotonic function means the dispatch will be efficient. We have further demonstrated that procurement costs when generators optimally bid in this market are below what they would be had they truthfully revealed costs. As for future work in this area, we hope to expand our analysis of joint auctions for energy and reserves with opportunity cost payments for reserves in a network setting with locational prices due to congestion. We will also explore the effect of differential ramp rate which may alter the order in which generators are deployed for energy production.

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